Measuring the Surface Tension of Water using the Pendant Drop Method*

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This experiment involves a simple method to determine the surface tension of a liquid, which in our case is water, by analyzing a drop formed at the edge of a tube containing the liquid. This technique is called the Pendant Drop Method since the liquid drop assumes a pendant shape under the influence of gravity. We will learn what surface tension is, its implications and how the dimensions of the pendant drop allow us to estimate its value. What is fascinating in this experiment is that the shape and form of the drop can provide quantitative insight about an important physical property.

KEYWORDS

Surface Tension · Young-Laplace Equation · Pendant Drop · Capillary Action · Cohesive and Adhesive Forces

1 Conceptual Objectives

In this experiment, we will,

1. understand how a liquid tends to assume the shape with least surface area,
2. understand how different forces cause the drop to assume a spherical shape,

3. understand that the determination of the morphology (shape and size) of the drop is one of the simplest ways to estimate the value of surface tension, and

4. learn how image analysis can be helpful in measuring useful physical properties.

2 Introduction

If the volume of an object is kept constant, the shape that it assumes to keep the smallest possible surface area is a sphere. In this context it is interesting to notice that a water droplet is spherical if there are no external forces acting on it. This phenomenon is an implication of **surface tension**—which is defined as **the elastic tendency of a fluid which makes it acquire the least surface area.**

The molecules in a liquid experience forces of attraction from other molecules, which we call **cohesive forces**. To understand the concept of surface tension in terms of forces, consider a molecule in the centre of a liquid body. Since it is surrounded by molecules from all sides, the cohesive forces cancel out. A molecule on the surface, however, experiences cohesive forces from molecules underneath it and from molecules on it's sides. The **adhesive forces** that it experiences from air molecules above are too weak compared to these cohesive forces [2]. Therefore, the surface molecules tend to contract to the minimal area, which of course, is a sphere.

Mathematically, if we increase the surface of an object by dA, we need to do extra work dW on it. The relationship between these two quantities is the following.

$$dW = \gamma dA \tag{1}$$

where γ is the surface tension.

Q 1. What are the units of surface tension?

Q 2. Calculate the work required to create a spherical drop of radius r.

Q 3. Calculate the work required to stretch a film of liquid up a height h. The length of the film is l and the surface tension is γ. Neglect gravitational effects.

2.1 The Pendant Drop

When a tube is filled with a liquid, which is then allowed to gradually drain, a small drop may form at the edge. You must have seen such a drop hung from the tip of

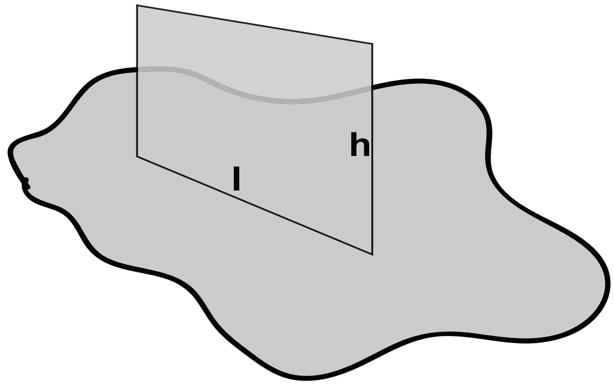

Figure 1: A thin film of area $h \times l$ created from a liquid.

a water faucet. We discussed earlier that a water drop tends to assume a spherical shape. In this case however, gravity elongates the hanging drop and stretches it into a pendant shape. The formation of this drop is explained by the fact that the surface tension forces cancel out the gravitational force acting on the drop.

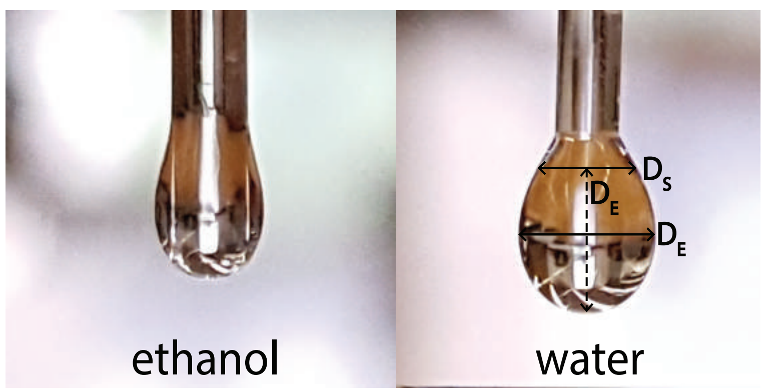

Figure 2: Typical ethanol and water pendant drops.

Now that you have learnt about surface tension, you are likely to realise how it causes many phenomena that we observe everyday. Certain insects take advantage of the high surface tension of water to walk on it! These insects are called **water striders**. In fact, a very simple experiment can further help you visualise this concept. Take a paper clip and gently place it over water. The paper clip will float, despite the fact that it is made of a material that is denser than water. If you dip it into the water, it will sink. This is explained by the fact that as long as it is above the water surface, the gravitational pull is balanced by the surface tension.

Surface tension is also the cause of capillary action and the presence of meniscuses.

If the surface tension of a liquid is smaller than that of a solid, it will tend to wet the solid. In the case of a tube, the liquid will tend to rise at the edges, forming a concave meniscus, as shown in the following figure. This experiment invites you to learn even more about this area in fluid mechanics. Some terms you could explore, and relate to surface tension are **surface free energy**, the **Young-Laplace Equation** and **wettability**. Check the references at the end of this manual.

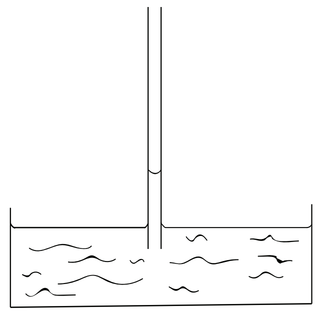

Figure 3: Water rising inside a glass tube due to capillary action.

2.2 Analysing the Pendant Drop

By using the fact that the surface tension forces and gravitational force cancel out for a pendant drop, an equation can be derived which describes the shape of this drop. This is known as the **Young-Laplace Equation**. The advanced method for calculating surface tension involves the iterative fitting of the Young-Laplace Equation onto the silhouette of the pendant drop. This process requires computational analysis. Another simpler way to estimate the surface tension is by capturing an image of the pendant drop, and measuring it's dimensions that can be directly plugged into the following formula [1]

$$\gamma = \frac{\Delta \rho g D_E^2}{H} \tag{2}$$

where γ is the surface tension in N/m, $\Delta\rho$ is the difference in density between the liquid and the air, g is the acceleration due to gravity and D_E is the maximum diameter of the pendant drop. Furthermore, H is a dimensionless function represented by the following equation.

$$\frac{1}{H} = f\left(\frac{D_S}{D_E}\right) \tag{3}$$

where D_S is the diameter of the pendant drop at a distance D_E from the bottom edge of the drop. For the purpose of this experiment, this function can be approximated by the following formula.

$$\frac{1}{H} = a\left(\frac{D_S}{D_E}\right)^b \tag{4}$$

where $a \approx 0.345$ and $b \approx -2.5$.

Q 4. What would be the value of D_S if the drop was a perfect sphere?

Q 5. Which external conditions does $\Delta\rho$ depend upon?

Substance	Surface Tension (mN/m)
ethanol	22.4
chloroform	26.7
water	72.8
mercury	486

Table 1: The surface tensions of various substances at 25°C

3 The Experiment

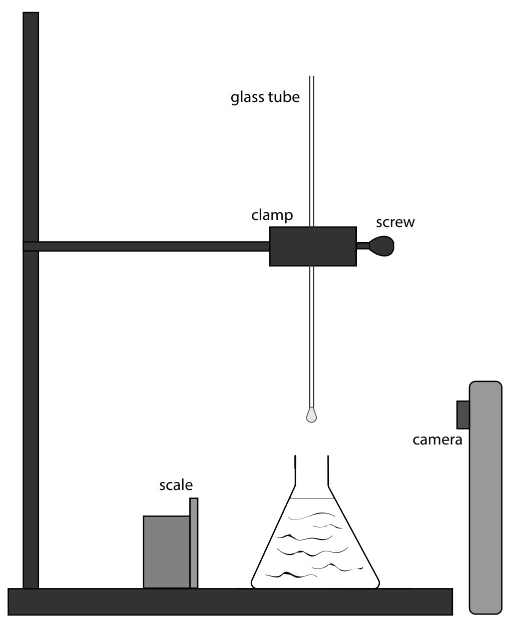

Figure 4: Setup of the experiment.

You will be provided with a thin glass tube, a stand and clamp, a beaker with water, and a meter rule fixed to a small piece of wood.

Place a glass tube into the clamp and **tighten the screw very slowly until the tube is held only gently held by the clamp**. A slight excess force will break the glass tube.

Caution: The glass tubes are very fragile. Do not keep you're eyes too close to them while performing the experiment. In case it breaks, do not touch any sharp edges.

Once the tube is in place, place the beaker directly underneath it. Slightly loosen the screw and dip the tube into the liquid down to a certain depth. Gently press the open end of the tube with your finger and take the tube out of the beaker. Gently tighten the screw again. Gradually remove your finger to let the liquid drain. At a certain point, you will observe the formation of a pendant drop which will fall in case of a slight disturbance. In case the all the liquid drains and a pendant drop does not form, repeat the procedure. This time, vary the depth till which you dip the tube into the liquid. You will just have to practice until you achieve a pendant drop.

Important: Make sure that the pendant drop is the largest possible drop that could have been formed. By repeating this procedure a few times, you will have an idea of how large the drop can be and you will be able to recognise a drop that is not properly formed. Such drops will not be used for analysis.

Place the scale as close to the pendant drop as possible. Use your smartphone camera or any other high resolution camera to capture an image of the drop. The image should be as clear as possible and both the pendant drop and the scale should be in focus. Transfer this image to the computer for analysis.

3.1 Analysis

1. Start the **Imagej** software.
2. Go to `File` → `Open` and chose your image.
3. Once the image has been loaded, go to `Process` → `Find Edges`.
4. If you further wish to make the image more clear, adjust the contrast by going to `Image` → `Adjust` → `Brightness/Contrast`.
5. Zoom into the scale and use the `Line Tool` to draw a line representing 1 cm.
6. Go to `Analyse` → `Set Scale`, and set this length to 1 cm.
7. Observe the pendant drop and locate it's maximum diameter. Use the `Line Tool` to draw this diameter. Make sure that you zoom in and extend this line to the **centre of the edges of the drop**. Go to `Analyse` → `Measure`, and

note down this length up to three decimal places. This is the value of D_E. Go to `Edit` and click on `Draw` to make this line permanent.

8. Note down the uncertainty in the D_E, and label the maximum and minimum values as $D_{E_{\max}}$ and $D_{E_{\min}}$.

9. Use the `Line Tool` to separately draw $D_{S_{\max}}$ and $D_{S_{\min}}$.

Use the following equations to calculate the values for D_E and D_S that we will use further in the experiment.

$$D_E = \sqrt{D_{E_{\max}} D_{E_{\min}}} \tag{5}$$

$$D_S = \sqrt{D_{S_{\max}} D_{S_{\min}}} \tag{6}$$

Q 6. What is the uncertainty in the reference scale? State it in the form $(1.000 \pm a)$ cm.

Q 7. What is the uncertainty in D_E and D_S? (**Hint:** Take into account the uncertainty due to the blurred edges of the pendant drop image **and** the uncertainty due to the scale from the previous question.)

Repeat the analysis with 5 drops so that you acquire five sets of values of D_E and D_S.

Q 8. State the values of D_E and D_S for each drop along with their total uncertainties.

Q 9. Use each set of values to calculate the surface tension and state its uncertainty in N/m.

Q 10. What could be the reason behind the high surface tension of water?

References

[1] Nicolas-Alexandre Goy, Z. Denis, M. Lavaud, A. Grolleau, N. Dufour, A. Deblais, U. Delabre. *The Physics Teacher*, pp. 498, vol. 55, (2017).

[2] Hewitt, Paul G. *Conceptual Physics*, pp. 261, 10th ed, (2006).