Name:……………………………………………………….

Total Marks:…………………………………………

GCSE (9-1) Grade 8/9

PROOF

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Show all your working out**

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets.
  – use this as a guide as to how much time to spend on each question.
- Questions labelled with an **asterisk** (*) are ones where the quality of your written communication will be assessed

Advice

- Read each question carefully before you start to answer it
- Attempt every question
- Check your answers if you have time at the end

1. Tanaka says 'When you multiply an odd number and an even number together, you will always get an odd number'.

Show that Tanaka is wrong.

(Total 2 marks)

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2. Tarish says,

'The sum of two prime numbers is always an even number'.

He is **wrong**.
Explain why.

....

....

(Total 2 marks)

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3. The n th even number is $2n$.
The next even number after $2n$ is $2n + 2$.

(a) Explain why.

....

....

(1)

(b) Write down an expression, in terms of n, for the next even number after $2n + 2$.

....

(1)

(c) Show algebraically that the sum of any 3 consecutive even numbers is always a multiple of 6.

(3)

(Total 5 marks)

4. Here are the first 4 lines of a number pattern.

$1 + 2 + 3 + 4$	$=$	$(4 \times 3) - (2 \times 1)$
$2 + 3 + 4 + 5$	$=$	$(5 \times 4) - (3 \times 2)$
$3 + 4 + 5 + 6$	$=$	$(6 \times 5) - (4 \times 3)$
$4 + 5 + 6 + 7$	$=$	$(7 \times 6) - (5 \times 4)$

n is the first number in the nth line of the number pattern.

Show that the above number pattern is true for the four consecutive integers n, $(n + 1)$, $(n + 2)$ and $(n + 3)$.

(Total 4 marks)

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5. n is a whole number.

Prove that $n^2 + (n + 1)^2$ is always an odd number.

(Total 2 marks)

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6. n and a are integers.

Explain why $(n^2 - a^2) - (n - a)^2$ is always an integer.

(Total 2 marks)

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7. n is an integer greater than 1.

Use algebra to show that $(n^2 - 1) + (n - 1)^2$ is always equal to an even number.

(Total 4 marks)

8. Prove that the difference between the squares of any two consecutive even numbers is always an odd number multiplied by 4.

(Total 4 marks)

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9. Prove algebraically that the sum of the squares of two consecutive integers is always an odd number.

(Total 3 marks)

***10.** Prove that the sum of the squares of any two odd numbers is always even.

(Total 4 marks)

11. Show that $(n + 3)^2 - (n - 3)^2$ is an even number for all positive integer values of n.

(Total 3 marks)

***12.** Prove that

$$(7n + 3)^2 - (7n - 3)^2$$

is a multiple of 12, for all positive integer values of n.

(Total 3 marks)

***13.** Prove algebraically that the product of two odd numbers is **always** an odd number.

(Total 3 marks)

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14. Prove algebraically that the sum of any two odd numbers is even.

(Total 3 marks)

***15.** Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \quad \text{is an even number}$$

for all positive integer values of n.

(Total 3 marks)

***16.** Given that *a* and *b* are two consecutive even numbers, prove algebraically that

$$\left(\frac{a+b}{2}\right)^2 \text{ is always 1 less than } \frac{a^2+b^2}{2}.$$

(Total 5 marks)

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17. Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4, for all positive integer values of n.

(Total 3 marks)

***18.** Prove that the sum of the squares of two consecutive odd numbers is never a multiple of 8.

(Total 4 marks)

19. Prove that

$$(2n + 3)^2 - (2n - 3)^2 \text{ is a multiple of } 8$$

for all positive integer values of n.

(Total 3 marks)

20. Prove, using algebra, that the sum of two consecutive whole numbers is always an odd number.

(Total 3 marks)

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21. n is an integer
Prove algebraically that the sum of $n(n+1)$ and $n+1$ is always a square number.

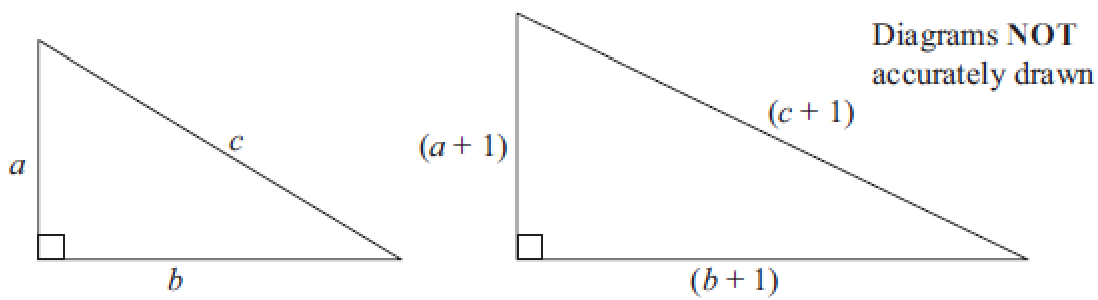
(Total 3 marks)

22. Umar thinks $(a+1)^2 = a^2 + 1$ for all values of a.

(a) Show that Umar is wrong.

(2)

Here are two right-angled triangles.
All the measurements are in centimetres.



(b) Show that $2a + 2b + 1 = 2c$

(3)

a, b and c cannot all be integers.

(c) Explain why.

(1)

(Total 6 marks)

***23.** Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

(Total 4 marks)

24. Prove that $(2n + 3)^2 - (2n - 3)^2$ is always a multiple of 12, for all positive integer values of *n*.

(Total 3 marks)

25. Prove that the sum of 3 consecutive odd numbers is always a multiple of 3

(Total 3 marks)

26. Prove algebraically that the sum of the squares of any 2 even positive integers is always a multiple of 4

(Total 3 marks)

27. Prove that the sum of the squares of 2 consecutive odd numbers is always 2 more than a multiple of 8

(Total 3 marks)

28. Prove algebraically that the sums of the squares of any 2 consecutive even number is always 4 more than a multiple of 8

(Total 3 marks)

29. The product of 2 consecutive positive integers is added to the larger of the two integers.

Prove that the result is always a square number

(Total 3 marks)

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30. c is a postive integer

Prove that $\frac{6c^3 + 30c}{3c^2+15}$ is an even number

(Total 4 marks)

TOTAL FOR PAPER IS 100 MARKS