Can Price Dispersion be supported solely by Information Frictions?

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Abstract

Yes, but one needs to assume that consumers know the realized price distribution, and that they do not know which firm has what price. Even with identical consumers and identical firms, if firms set prices in a first stage, and if consumers search sequentially in a second stage, then price dispersion arises in the form of a mixed strategy subgame perfect Nash Equilibrium. In contrast to Burdett and Judd (1983), price quotes are not required to be "noisy." Moreover, actual search is predicted to be nontrivial. (*JEL* L13, D83, D21)

Keywords: Price dispersion; information frictions; sequential search.

1 Introduction

In his economics of information article, Stigler (1961) wrote that "it would be metaphysical, and fruitless, to assert that all [price] dispersion is due to heterogeneity." Since then, several studies have confirmed the empirical significance of price dispersion.[1] However, after Stigler's seminal paper, Diamond (1971) presented a challenge: If firms and consumers are identical, and if consumers pay to sequentially search for prices, the only Nash equilibrium is the monopoly price. Intuitively, if

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[1]See, for example, Kaplan and Menzio (2015), Hortaçsu and Syverson (2004), or Sorensen (2000).

all firms set the monopoly price, the consumer will not search, but if the consumer will not search, all firms set the monopoly price. As Reinganum (1979) said, the Diamond paradox seemed to imply that "imperfect information alone is insufficient to support price dispersion."

This paper adds to recent efforts by showing with minimal assumptions that imperfect information alone is indeed sufficient to support price dispersion. I assume that firms fix their prices *before* consumers search, and that consumers know the distribution of actual prices being charged in the market, but do not know which firm is charging which price. This paper shows that this information structure is sufficient to generate price dispersion, even with homogeneous firms and homogeneous consumers.

Intuitively, in the Diamond model, firms cannot profit from a reduction in prices, because firms and consumers choose their strategies simultaneously. However, the passivity of the consumer is broken when we unfold the model into two stages, because her strategy now becomes a complete contingent plan. Hence, firms anticipate the consumer's reaction to a cut in prices, and thus have incentives to steal the market instead of sharing it. The main result is that, even if we have no a priori reason to expect ex post price heterogeneity, monopoly pricing is not subgame perfect, and price dispersion arises in the form of a mixed strategy subgame perfect Nash equilibrium. Finally, this paper overcomes "a host of technical problems that haunt [this information structure], e.g. troublesome integer conditions (typically overlooked) in constructing dispersed equilibria" (Stahl II, 1996), which is a strong reason why the literature has favored other information structures.

This paper is related to the search literature interested in obtaining price dispersion under minimal assumptions.[2] For example, Burdett and Judd (1983) achieve price dispersion by allowing price quotes to be stochastic. However, in contrast with my paper, they predict no actual search in equilibrium.

Perhaps the closest paper is Menzio and Trachter (2015) where buyers search sequentially among a continuum of small firms and a single large firm. In their model, price dispersion arises as equilibrium mixed strategies, and price dispersion is obtained by exploiting the heterogeneity between small firms and a single large

[2]See Rauh (2007), which considers heterogeneity in search costs, demand functions and production functions, thus subsuming most of the literature as special cases. Current theoretical research focuses on the Stahl II (1989, 1996) and Wolinsky (1986) models which lend themselves easily to structural estimation.

firm. In some sense, this paper shows that the result in Menzio and Trachter (2015) can be extended to a market composed of identical large firms, where dispersion is obtained through the informational assumption while maintaining ex ante homogeneity.

2 The model

The model has two stages. In the first stage, n identical firms produce an homogeneous good, and simultaneously fix their prices $p_i \in \mathcal{P}_i$, $i = 1, 2, \ldots, n$, where $\mathcal{P}_i$ may be either continuous or discrete. I assume no production costs.[3] After prices are chosen, the second stage begins, and a unique consumer enters the market.

The consumer has a unit demand, and a willingness to pay normalized to 1. That is, she will never pay more than 1 to buy the good. To be able to buy, the consumer must pay a fixed search cost s each time she arrives at a store. The first search is free.[4] In each search, the consumer randomly arrives at one of the n stores with equal probability. There, she must decide whether she buys the good and exits the market, or rather continues her search. Sampling is with replacement.[5]

To be consistent with random search, suppose that the consumer has no recall (assuming perfect recall yields the same results).[6] Note also that $s = 0$ is equivalent to perfect information, as the consumer could search forever.

Finally, the structure of the model and the rationality of the consumer and firms are common knowledge.

The informational assumption that drives the result is that the consumer knows the price distribution when she starts searching the market. Note that, at this second stage of the game, such distribution is a realization of the equilibrium price distribution. As Menzio and Trachter (2015) explain, "This is the same assumption made in the vast majority of search models where the price distribution is exogenously given (e.g., Stigler, 1961; McCall, 1970). Indeed, only a few papers model the buyer's problem of learning about the price distribution while searching (see, e.g.,

[3] Janssen, Pichler and Weidenholzer (2011) consider stochastic production costs.

[4] Ellison and Wolitzky (2012) consider firms that can increase the search cost by obfuscating. Janssen, Moraga-González and Wildenbeest (2005) consider a costly first search.

[5] Here, I follow Carlson and Mcafee (1983), because sampling without replacement is considerably harder to analyze.

[6] Daughety and Reinganum (1992) consider endogenous recall.

Rothschild, 1974; Burdett and Vishwanath, 1988). The assumption is also common in models where prices are endogenous (see, e.g., Pratt, 1979; Rob, 1985; Stiglitz, 1987). Other models, such as Reinganum (1979), Burdett and Judd (1983) and Stahl II (1989), assume that buyers do not observe the actual price distribution, but have rational expectations about its equilibrium value." For example, in a heterogeneous production cost model, the consumer is assumed to know about the distribution of production costs in order to compute the equilibrium price distribution. However, this assumption seems as strong as the one used on this paper, in particular in an extensive form game. I thus motivate my assumption by thinking of consumers that learn the price distribution by reading the newspaper or searching online before engaging in physical search. For the modern consumer, who is connected to the internet, this assumption is entirely plausible. In fact, a popular US price-comparison website in the automobile market literally shows the actual price distribution to consumers.

2.1 Optimal search

The goal of the consumer is to buy at the lowest price, so at each period she chooses between either buying at the quoted price, or paying the search cost and going to another store at random. The marginal cost of the extra search is s at any point. The marginal benefit given the current price at hand, p, is the expected discount $\frac{1}{n}\sum_{j=1}^{n}(p-p_j)\mathbb{1}\{p_j \leq p\}$, where $\mathbb{1}$ is the indicator function. A *reservation price*, $p^r(\mathbf{p})$, is a function of the vector of prices in the economy, $\mathbf{p} = (p_1, \ldots, p_n)$, that sets the marginal benefit equal to the marginal cost of the extra search. Appendix Lemma 5 shows that such a price exists, and is implicitly defined by

$$p^r(\mathbf{p}) = s + \frac{1}{n}\sum_{j:p_j \leq p^r(\mathbf{p})} p_j + \frac{\#\{j : p_j > p^r(\mathbf{p})\}}{n} p^r(\mathbf{p}), \tag{1}$$

where # is the cardinality of a set, and the dependence of p^r on s is omitted.

The optimal stopping rule and ending of the model is the standard condition found in search models: stop searching when $p \leq p^r(\mathbf{p})$.[7] Let R^* denote this optimal stopping rule, which is a dominant strategy. Also, let $\mathcal{R}$ denote the space of strategies for the consumer's stopping rules. Thus,

[7]Note that the optimal search rule is myopic (Weitzman, 1979).

Lemma 1. The consumer always follows R^*.

2.2 Optimal firm's strategy

Let the profits of firm i be denoted by $\pi_i(\mathbf{p}, R)$, which is a function of prices and the stopping rule of the consumer. Assume firms are risk neutral. Then, because of Lemma 1, profits can be stated as:

$$\pi_i^*(p_i, p_{-i}, R^*) = \begin{cases} p_i & \text{if } p_i \leq p^r(\mathbf{p}) \text{ and } p_j > p^r(\mathbf{p}) \ \forall j \neq i \\ \dfrac{p_i}{k} & \text{if } p_i \leq p^r(\mathbf{p}) \text{ and } p_j \leq p^r(\mathbf{p}) \\ & \quad \forall j \in J \text{ with } \#J = k-1 \\ 0 & \text{in any other case} \end{cases} \tag{2}$$

where p_{-i} is the vector of prices p_j, with $j \neq i$, and $J = \{j | p_j \leq p^r(\mathbf{p})\}$. Using notation from Reny (1999), the game that the firms and the consumer play is defined next.

Definition 1 (Game 1)**.** The game in extensive form that the n firms and the consumer play is given by $\{(\Delta\mathcal{P}_i, \pi_i(\cdot))_{i=1}^n, (\Delta\mathcal{R}, -L)\}$, where n is the number of firms; $\mathcal{P}_i$ is the strategy space for each firm; $\mathcal{R}$ is the strategy space for the consumer's stopping rules; π_i is firm i's profit function; and L is the total payment made by the consumer.

Because the firms will never choose a price above 1, henceforth consider two cases: (1) a continuous $\mathcal{P}_i = [0, 1]$ for all i; or (2) a discrete $\mathcal{P}_i = \left\{0, \frac{1}{v}, \frac{2}{v}, \ldots, 1\right\}$, where $v + 1$ is the number of grid points.

Define price dispersion as a nontrivial mixed strategy that arises in equilibrium. We are interested in the cases where *pure* strategies equilibria do not exist, but *mixed* strategies equilibria do. The following theorem shows that Game 1 always has a Nash equilibrium, and that the equilibrium must be mixed if some conditions are met. First, I define such conditions.

Definition 2 (Price dispersion conditions)**.**

- For continuous pricing: $s > 0$ and $ns < \frac{n-1}{n}$.
- For discrete pricing: $s \geq \frac{n-1}{n} \cdot \frac{1}{v}$ and $\left(ns, \frac{n-1}{n}\right) \cap \mathcal{P}_i \neq \emptyset$.

Theorem 3 (Price dispersion)**.** Game 1 always has a subgame perfect Nash equilibrium. Moreover, Game 1 has no subgame perfect Nash equilibria in *pure* strategies if and only if the conditions in Definition 2 hold. That is, Game 1 exhibits price dispersion if and only if the conditions in Definition 2 hold.

Theorem 3 formally discards *pure* strategy equilibria from the game under some "Goldilocks" conditions: an intermediate range where the search cost is not too low and not too high. Intuitively, a small search cost amounts to perfect information, which yields competition à la Bertrand. On the other hand, if the search cost is too high, firms enjoy market power, because information is too expensive for the consumer, which results in a captive costumer. Thus, a high search cost yields the Diamond equilibrium. Finally, and aligned with the intuition of Stahl II (1989), costs accumulate when searching among many firms. Therefore, the search cost must be weighted by the number of firms in the market.

In the proof, I show that Game 1 always has a subgame perfect Nash equilibrium regardless of the set of parameters. However, if the search cost meets with the the theorem's conditions, then the equilibrium must be in mixed strategies for the firm.

2.3 Proof of Theorem 3

First, I prove existence of a subgame perfect Nash equilibrium for Game 1.

If $\mathcal{P}_i$ is discrete for each i, the sets $\mathcal{P}_i$ have finite number of elements. Furthermore, as the consumer always plays R^*, it follows that the subgame of Game 1 that takes strategy R^* as given—call it Γ^*—is a finite strategic form game. Therefore, it has a mixed strategy Nash equilibrium. Corollary 7 characterizes the equilibrium.

On the other hand, the case where $\mathcal{P}_i$ is continuous presents serious challenges, because the payoffs $\pi_i^*(p_i, p_{-i}, R^*)$ are not continuous in (p_i, p_{-i}) nor quasiconcave in p_i. It follows that the convexity and upper hemicontinuity of the best response correspondence cannot be assured. Therefore, the existence of a mixed strategy Nash equilibrium cannot be obtained with standard arguments. Fortunately, Reny (1999) establishes conditions under which equilibria exist in a discontinuous game.

Let q_i be the mixed strategy of firm i and $\mathbf{q} = (q_1, \ldots, q_n)$. Formally, $q_i \in \Delta\mathcal{P}_i$ is a probability measure on Borel subsets of $\mathcal{P}_i$, $\mathcal{B}(\mathcal{P}_i)$. Moreover, $\Delta\mathcal{P}_i$ is a compact metric space when endowed with the Prohorov metric (Billingsley, 1999). Also, define Γ^* as the subgame of Game 1 where the consumer plays R^*, $\pi_i^*(\mathbf{q}) \equiv$

$\int_{\mathcal{P}} \pi_i^*(\mathbf{p}, R^*) d\mathbf{q}$ for all $\mathbf{q} \in \Delta\mathcal{P} \equiv \times_i \Delta\mathcal{P}_i$, and endow all product sets with the product topology.

To show that a Nash Equilibrium in mixed strategies exists, we need to show that the mixed extension of Γ^* is *better-reply secure*, as in Reny (1999).[8] I conduct the proof in a series of steps. First, I show that the sum of the payoffs is upper semicontinuous in Lemma 2. Second, I prove a property that implies that the game is payoff secure in Lemma 3. Third, combine Lemmas 2 and 3 to show that the game is better-reply secure, and conclude that it must have a Nash Equilibrium.

Lemma 2. $\sum_{i=1}^{N} \pi_i^*(\mathbf{p})$ is upper semicontinuous in $\mathbf{p}$ on $\mathcal{P}$.

Proof. By definition, $\sum_{i=1}^{n} \pi_i^*(\mathbf{p})$ is equal to the mean of the prices that are equal or less than the reservation price. Fix any $\mathbf{p}$ and any $\varepsilon > 0$. Notice that increasing any element of $\mathbf{p}$ by some small amount $\delta > 0$ will either increase $\sum_{i=1}^{n} \pi_i^*(\mathbf{p})$ by δ at the most, or decrease $\sum_{i=1}^{n} \pi_i^*(\mathbf{p})$, because some $p_i + \delta$ might become higher than $p^r(\mathbf{p})$. Notice also that decreasing any element of $\mathbf{p}$ by some small amount $\delta > 0$ will unambiguously decrease $\sum_{i=1}^{n} \pi_i^*(\mathbf{p})$ by two channels: (1) because the average of prices decreases; and (2) because low prices being reduced may cause some of the high prices to become higher than $p^r(\mathbf{p})$. Therefore, there is always a neighborhood $N_{\delta(\varepsilon)}(\mathbf{p})$ such that

$$\sum_{i=1}^{n} \pi_i^*(\mathbf{p}') \leq \sum_{i=1}^{n} \pi_i^*(\mathbf{p}) + \varepsilon \qquad \forall \mathbf{p}' \in N_{\delta(\varepsilon)}(\mathbf{p}).$$

$\therefore \sum_{i=1}^{N} \pi_i^*(\mathbf{p})$ is upper semicontinuous in $\mathbf{p}$ on $\mathcal{P}$. □

Lemma 3. The game Γ^* satisfies the following property: For all $i = 1, \ldots, n$, $\varepsilon > 0$, $p_i \in \mathcal{P}_i$ and $q_{-i} \in \Delta\mathcal{P}_{-i}$, there exists $\widehat{p}_i \in \mathcal{P}_i$ such that

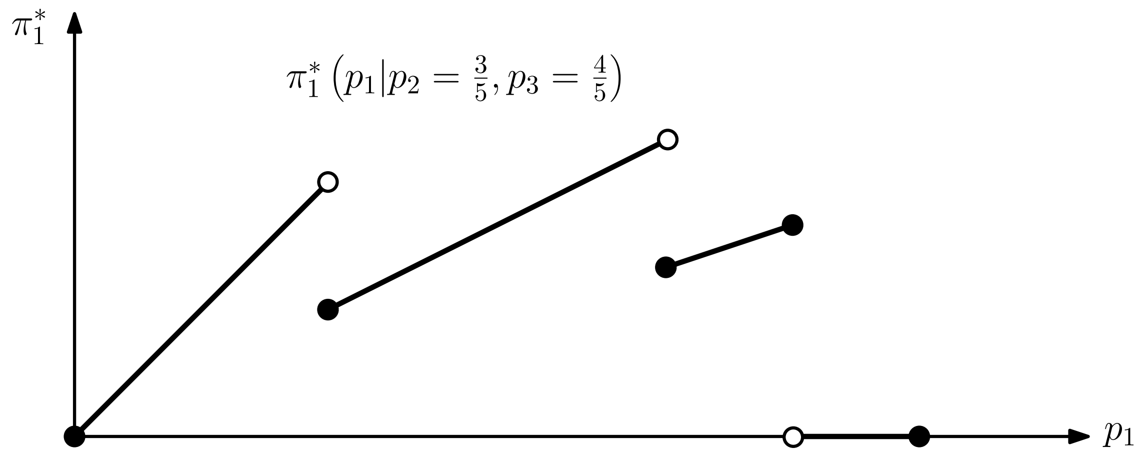
$$q_{-i}\Big(\{p_{-i} \in \mathcal{P}_{-i} : \pi_i^* \text{ is discontinuous at } (\widehat{p}_i, p_{-i})\} \Big) = 0$$

and $\pi_i^*(\widehat{p}_i, q_{-i}) \geq \pi_i^*(p_i, q_{-i}) - \varepsilon$.

Proof. Note that π_i^* is discontinuous in a countable set. Moreover, i can always find a deviation $\widehat{p}_i$ that is slightly worse than p_i but satisfies $\pi_i^*(\widehat{p}_i, q_{-i}) \geq \pi_i^*(p_i, q_{-i}) - \varepsilon$.

[8] For the definitions of better-reply security and payoff security, please refer to Reny (1999). The reader might ask if Game 1 is better-reply secure. It is not. Consider $n = 2$, $s = 1/10$, $p_1 = 2/5$ and

A deviation such as $\widehat{p}_i \equiv p_i - \delta(\varepsilon)$ works for a well chosen $\delta(\varepsilon)$. See Figure 1 for reference. Finally, such deviation can always be chosen away from those discontinuity points. □



Note: Qualitatively, all payoff functions are the same. Discontinuities are from the left and "jump down" from the left with the exception of the last segment, which becomes irrelevant.

FIGURE 1: Example of a payoff function with 3 firms and $s = 0.5$.

As the last step, combining Lemma 2 and Proposition 5.1 in Reny (1999), we get that Γ^* is reciprocally upper semicontinuous. Combining Lemma 3 with Theorem 3.33 in Carmona (2013), we get that Γ^* is payoff secure. Finally, by Proposition 3.2 in Reny (1999), Γ^* is better-reply secure.

To complete the existence part of Theorem 3, by Corollary 5.2 in Reny (1999) and better-reply security, Γ^* possesses a mixed strategy Nash Equilibrium.

In the second part of the proof, I show that the equilibrium cannot be in *pure* strategies if and only if the conditions of Definition 2 hold.

First, sufficiency. I will show that, for any $\mathbf{p} = (p_1, \ldots, p_n)$, there exists a firm that has incentives to deviate. The following lemma has the purpose of narrowing the strategies that may arise in a pure strategy equilibrium.

Lemma 4. Suppose that $s > 0$ and $(p_1, \ldots, p_n)$ are such that $p_i > p^r(\mathbf{p})$ for some i. Then the pure strategy profile $(p_1, \ldots, p_n, R^*)$ is not a subgame perfect Nash equilibrium for Game 1.

$p_2 = 3/5$ as a counterexample.

Proof. I will propose a deviation p_i^* such that $p_i^* \leq p^r(\mathbf{p}^*)$. If $\mathcal{P}_i$ is discrete, since $p_i > p^r(\mathbf{p})$ for some firm i, let $p_i^* = 1/v$ be a deviation. If $\mathcal{P}_i$ is continuous, let $p_i^* = \epsilon$ be a deviation with $0 < \epsilon < s$.

By way of contradiction, suppose that $p_i^* > p^r(\mathbf{p}^*)$. Then, $p_j \geq p^r(\mathbf{p}^*)\ \forall j$ and by equation (1), we have that $p^r(\mathbf{p}^*) = s + p^r(\mathbf{p}^*)$ which is a contradiction. Therefore, $p_i^* \leq p^r(\mathbf{p}^*)$ and $\pi_i^*(p_i^*, p_{-i}) > \pi_i^*(p_i, p_{-i}) = 0$. □

Because of Lemma 4, it suffices to review the following cases.

Case 1: $p_i < p^r(\mathbf{p})\ \forall i$

- Case 1.1: For some j, i with $j \neq i$, it is true that $p_j = \min_k\{p_k\} < \max_k\{p_k\} = p_i$. Consider the deviation $p_j^* = \max_k\{p_k\} = p_i$. Then, $p_j^* = p_i < p^r(\mathbf{p}) \leq p^r(\mathbf{p}^*)$, because $p^r(\mathbf{p})$ is nondecreasing in $\mathbf{p}$.
- Case 1.2: $p_i = p_j < 1\ \forall\ i, j$.

– If $\mathcal{P}_j$ is discrete, then consider the deviation $p_j^* = p_j + \frac{1}{v}$. By way of contradiction, suppose that $p_j^* > p^r(\mathbf{p}^*)$. Then, equation (1) implies that

$$\begin{aligned} p^r(\mathbf{p}^*) &= s + \frac{n-1}{n}p_j + \frac{p^r(\mathbf{p}^*)}{n} \\ \Leftrightarrow p^r(\mathbf{p}^*)\left(\frac{n-1}{n}\right) &= s + \frac{n-1}{n}p_j \\ \Leftrightarrow p^r(\mathbf{p}^*) &= \frac{s}{\frac{n-1}{n}} + p_j \\ \Rightarrow s &< \frac{n-1}{n}\cdot\frac{1}{v}. \qquad \text{(contradiction)} \end{aligned}$$

The contradiction implies that $p_j^* \leq p^r(\mathbf{p}^*)$. Thus, $\pi_i^*(p_i^*, p_{-i}) > \pi_i^*(p_i, p_{-i})$.

– If $\mathcal{P}_j$ is continuous, then consider the deviation $p_j^* = p_j + \epsilon$, such that $0 < \epsilon < s$ and $p_j^* \leq v$. Analogous to the discrete case, equation (1) implies that $s < \frac{n-1}{n}\epsilon$, which is a contradiction. Therefore, $p_j^* \leq p^r(\mathbf{p}^*)$ and $\pi_i^*(p_i^*, p_{-i}) > \pi_i^*(p_i, p_{-i})$.

- Case 1.3: $p_i = 1\ \forall i$.

  I will construct a deviation $p_j^* = p_j - \delta$ with $\delta \in \left(ns, \frac{n-1}{n}\right) \cap \mathcal{P}_j$ such that the following conditions are satisfied.

  1. $p_j^* \leq p^r(\mathbf{p}^*)$.
  2. $p_i^* > p^r(\mathbf{p}^*)\ \forall\ i \neq j$.
  3. $\pi_j^*(p_j^*) > \pi_j^*(p_j)$.

  Condition 1 is satisfied because $p^r(\mathbf{p})$ is nondecreasing (Lemma 5).

  Condition 2 requires firm j to win the whole market. Suppose, by way of contradiction, that $p_i \leq p^r(\mathbf{p}^*)$, for $i \neq j$. Recall that $p_i = 1$ for all i. Then,

$$p^r(\mathbf{p}^*) = s + \frac{1}{n}\sum_{i=1}^{n} p_i = s + 1 - \frac{\delta}{n} \geq p_i = 1 \Rightarrow \delta \leq ns. \qquad \text{(contradiction)}$$

The contradiction implies that $p_i^* > p^r(\mathbf{p}^*)\ \forall\, i \neq j$.

Finally, since firm j wins the entire market under the first two conditions, Condition 3 requires the firm to be better off:

$$\pi_j^*(p_j^*) > \pi_j^*(p_j) \Leftrightarrow 1 - \delta > \frac{1}{n} \Leftrightarrow \delta < \frac{n-1}{n},$$

but due to the hypothesis, such δ exists, and $p_j^* = p_j - \delta \in \mathcal{P}_j$. Therefore, p_j^* is a deviation.

Case 2: $p_i < p^r(\mathbf{p})$ and $p_j = p^r(\mathbf{p})$ for some $i \in I$ and some $j \in J$ such that $I \cup J = \{1, 2, \ldots, n\}$ and $I \cap J = \emptyset$. For any $i \in I$ consider the deviation $p_i^* = p_j > p_i$. Then, $p_i^* = p^r(\mathbf{p}) \leq p^r(\mathbf{p}^*)$, which implies that $\pi_i^*(p_i^*) > \pi_i^*(p_i)$.

Therefore, Game 1 has no subgame perfect Nash equilibria in pure strategies if the conditions of Definition 2 hold.

Finally, for necessity, note that if there are no subgame perfect Nash equilibria in pure strategies for Game 1, then there always exists a profitable deviation. Since $\mathbf{0}$ is not an equilibrium, this implies that $s > 0$ for the continuous case, or $s > \frac{n-1}{n} \cdot \frac{1}{v}$ for the discrete case. Moreover, going again through Case 1.3 of the proof and using a contradiction argument, it follows that $\left(ns, \frac{n-1}{n}\right) \cap \mathcal{P}_i \neq \emptyset$. Therefore, the conditions of Definition 2 hold.

As a corollary, it is straightforward to show that $s = 0$ implies a Bertrand equilibrium, and that the Diamond equilibrium arises if $s > 0$ and the conditions of Definition 2 do not hold. □

3 Equilibrium properties and discussion

In general, the equilibrium has no analytical solution. However, we can use the discrete pricing equilibrium as an arbitrarily good approximation of the continuous pricing equilibrium by increasing the number of grid points, v. For practical pur-

[9] A strategic approximation of the normal form game $(\mathcal{P}_i, \pi_i^*)_{i=1}^n$ in which the firms take R^* as given is a countable set of pure strategies $\mathcal{P}^\infty = \mathcal{P}_1^\infty \times \cdots \times \mathcal{P}_n^\infty$ contained in $\mathcal{P} = \mathcal{P}_1 \times \cdots \times$

poses, we can always think of the willingness to pay in cents of a dollar. Formally, we have a *Strategic Approximation* (Reny, 2011).[9]

Corollary 4. Suppose that $\mathcal{P}_i$ is continuous for each i. Then Game 1 has a Strategic Approximation given by the discrete pricing equilibrium.

Proof. The existence of a strategic approximation is immediate given that the game is better-reply secure, and by Theorems 1 and 2 in Reny (2011). Moreover, the discrete pricing equilibrium is an $\varepsilon-$equilibrium of the continuous pricing game. To see this, note that for a fixed ε, a fine enough grid can approximate any price with arbitrary precision. Because the payoff functions are semicontinuous[10], there exists $v(\varepsilon)$ large enough such that a price in the grid $\varepsilon-$approximates, either from above or below, the payoff of any price in [0,1]. Finally, limits of $\varepsilon-$equilibria are equilibria of the continuous pricing game, because the game is better reply secure (Reny, 1999). □

I henceforth focus on the discrete pricing equilibrium, which is characterized by Appendix Corollary 7 in a system of nonlinear equations.

Consider the following example. Let an economy consist of two firms, and $\mathcal{P}_i = \left\{0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right\}$. That is, discrete pricing with $n = 2$ and $v = 5$. Assume $s = 1/10$, which meets the conditions of Definition 2. Therefore, no pure strategy equilibria exist, but it can be shown that multiple symmetric mixed Nash equilibria exist (see Appendix B).

Figure 2 shows the distribution of prices played with positive probability when the grid increases from $v = 5$ to 10, 25 and 50 points. Remarkably, the equilibrium price distribution is bimodal. In this example, firms charge high prices almost all the time but, occasionally, we have a sale with 50% discount.

In general, some basic properties of the equilibrium can be obtained, even without analytical solutions. First, the price distribution shifts to the right when the search cost increases, yielding higher profits for firms, and lower surplus for the consumer. Second, holding fixed the absolute size of the search cost, increasing the

$\mathcal{P}_n$, such that whenever for each player i, $\mathcal{P}_i^1 \subseteq \mathcal{P}_i^2 \subseteq \cdots$ is an increasing sequence of finite subsets of $\mathcal{P}_i$ whose union contains $\mathcal{P}_i^\infty$, any limit of equilibria of the sequence of finite games $(\mathcal{P}_i^1, \pi_i^*)_{i=1}^n, (\mathcal{P}_i^2, \pi_i^*)_{i=1}^n, \ldots$ is an equilibrium of $(\mathcal{P}_i, \pi_i^*)_{i=1}^n$.

[10] The payoffs are upper semicontinuous in own prices over some segments, and lower semicontinuous over other segments (there are no isolated points on the graph, see Figure 1). Moreover, because $p^r(\mathbf{p})$ is continuous, the same is true with respect to the other firms' prices.

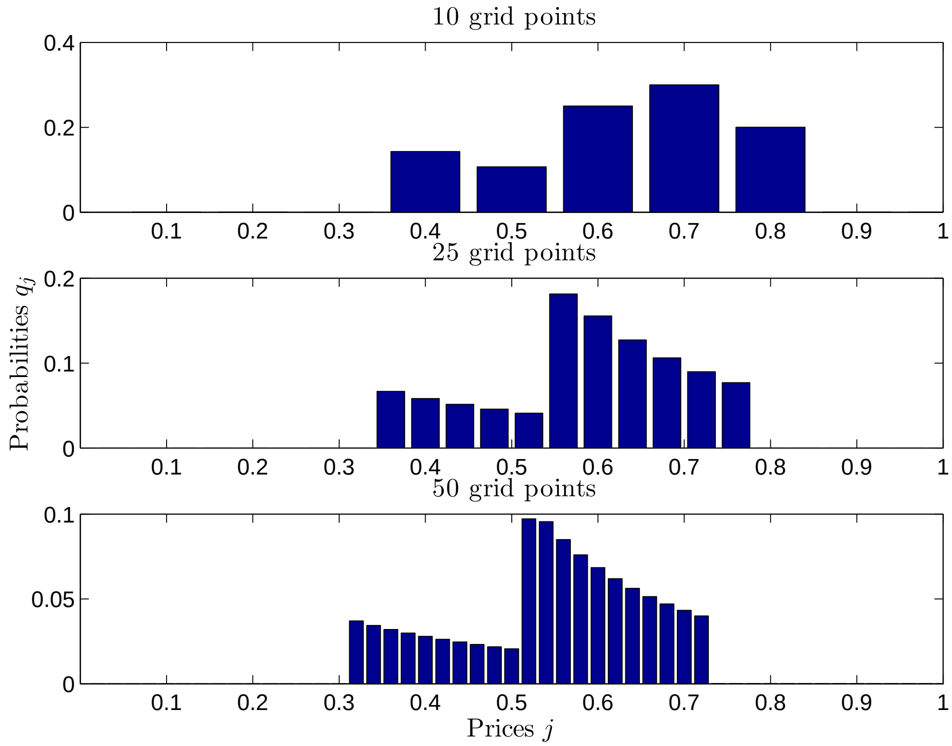


FIGURE 2: Equilibrium distributions with 2 firms and $s = 1/10$ for grids of size 10, 25 and 50.

number of firms results in monopolistic pricing. The following corollaries formalize the results. The discussion that follows illustrate them.

Corollary 5. In the symmetric equilibrium, the expected price is nondecreasing in the search cost. Moreover, the equilibrium payoff of the firms is also nondecreasing in the search cost, while the consumer surplus is nonincreasing in the search cost.

Proof. From Lemma 5, the reservation price is nondecreasing in the search cost. Then, from equation (2), $\pi_i(\mathbf{p}, R^*)$ is nonincreasing in s when $p_i \leq p^r(\mathbf{p})$ and $p_{-i} > p^r(\mathbf{p})$. Analogously, $\pi_i(\mathbf{p}, R^*)$ is nondecreasing in s in any other case. A Nash equilibrium requires i to be indifferent between choosing p_i and p_i' whenever both p_i and p_i' have positive weights in the equilibrium mixed strategy. It follows that strategies yielding $p_i \leq p^r(\mathbf{p})$ and $p_{-i} > p^r(\mathbf{p})$ must receive zero weight when the search cost is high enough. Then, the equilibrium weights shift to the right as s increases. Then, the expected price is nondecreasing. Since, the consumer faces weakly higher prices, her surplus weakly decreases. □

Corollary 6. For a fixed $s > 0$, as $n \to \infty$, the unique equilibrium is the Diamond equilibrium. That is, $p_i = 1$, for all i.

Proof. Follows immediately from Theorem 3. □

The next figures illustrate the effects of increasing the search cost and entry through simulation. I only consider sets of parameters that create price dispersion, because the discussion becomes trivial otherwise. All the results are qualitatively robust to different specifications of the parameters, and are selected for clarity of exposition.

Figure 3 considers different search costs when the grid size $v = 10$, and when the number of firms $n = 2$. Increasing the search cost has a clear effect: a higher search cost moves the price distribution to the right whence firms benefit from more monopolistic power.

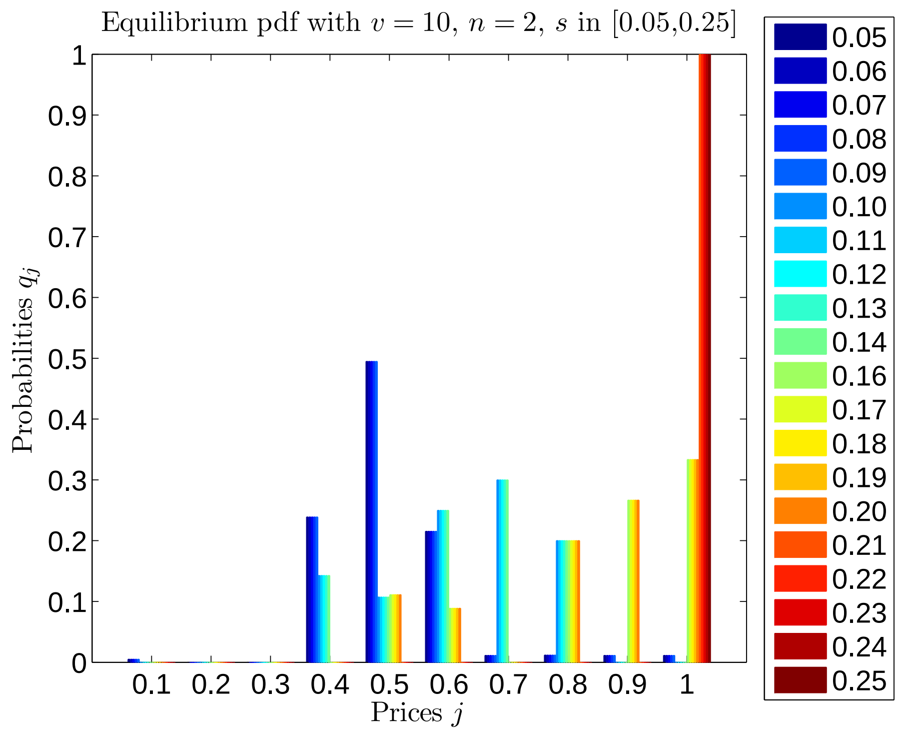


FIGURE 3: Comparative statics on the search cost.

In a simple model such as this one, the total surplus from the Bertrand and Diamond equilibria are the same, because there is no search in either equilibrium. Either the consumer or the producer extract all surplus, and no dead weight loss exists. In the Dispersed equilibrium, however, the search cost must be subtracted from the consumer surplus, which in turn decreases total surplus.

The producer surplus, π, is obtained with Corollary 7. Once the equilibrium distribution is obtained, one can construct the consumer surplus by Monte Carlo simulation.[11] Finally, total surplus is equal to the consumer surplus plus n times the producer surplus. Figure 4 summarizes these observations where the price discreteness causes the five-points pattern in which firms use the same strategy. As expected, the consumer is worse off when the search cost rises, but the producer is better off. Moreover, the total surplus shows the dead weight loss due to search for intermediate search cost levels.

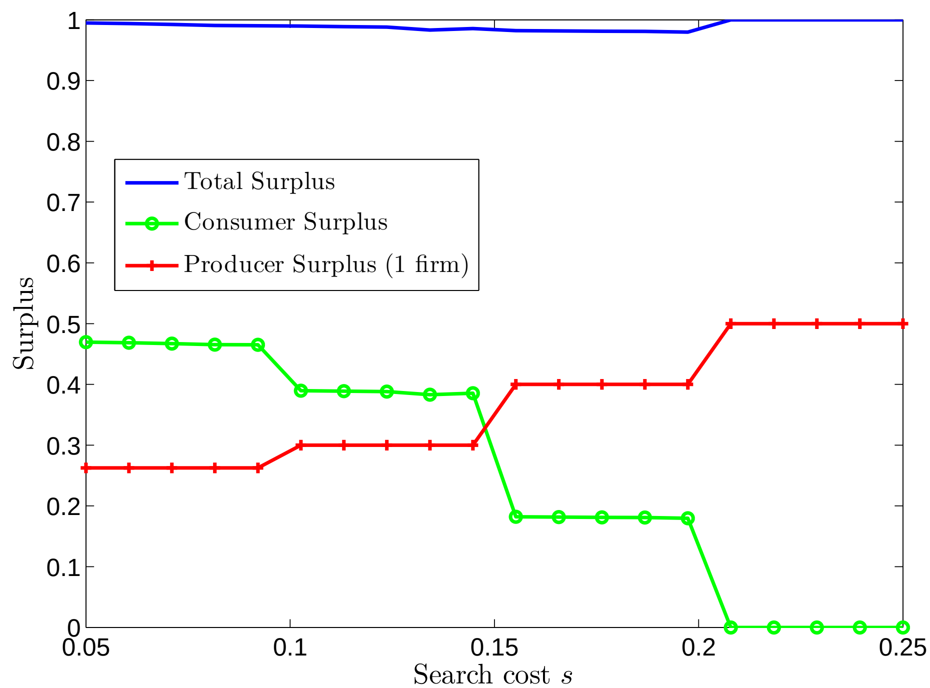


FIGURE 4: Welfare analysis. $v = 10$, $n = 2$ and $s \in [0.5, 2.5)$.

Finally, Figure 5 shows an example of the effect of increasing the number of firms. As more firms enter the market, the average price and the variance increase. Additionally, the equilibrium distribution shows nonmonotonic behavior on kurtosis,

[11]Simulate draws of equilibrium prices, construct the reservation price and calculate the expected number of searches and the expected price paid. The Matlab codes used for these calculations, and for those of Section 3, are available upon request or at `jtudon.com`.

which measures the tails of the distribution. In this case, the distribution has fatter tails with 3 firms, but is less dispersed than with 4 firms.

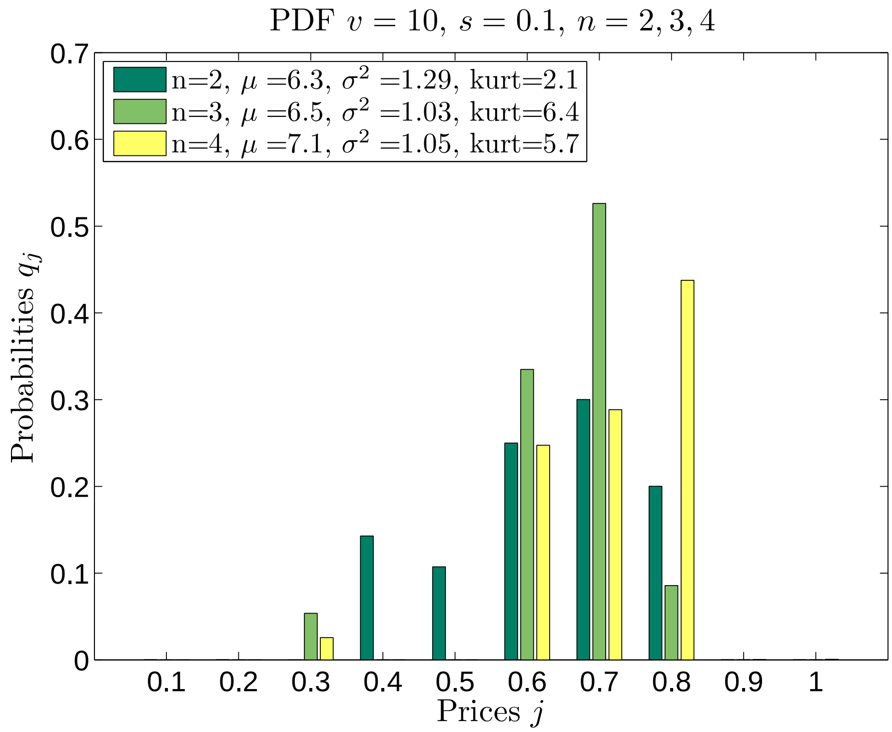


FIGURE 5: Increasing the number of firms

Admittedly, the noncompetitive effect of increasing the number of firms is a limitation, but is not new to search models; see Robert and Stahl II (1993), Stahl II (1989) or Rosenthal (1980) for example. The probability of being the lowest-priced store decreases exponentially with entry, disrupting the ability of firms to steal the market. However, while the condition $ns < (n-1)/n$ becomes rapidly stringent as the number of firms in the market grow, real life examples show that search costs can be within a Goldilocks zone. De los Santos, Hortaçsu and Wildenbeest (2013) estimate the search costs of consumers buying MP3 players. The authors consider up to 10 different firms offering several devices whose prices range considerably. Though their baseline model is different, their results show that the constraint is plausible.[12] Moreover, note that the result is not driven by a "small" n: even for 2 firms, if the

[12] Specifically, from Table 4 in De los Santos, Hortaçsu and Wildenbeest (2013) one can obtain a lower bound on the willingness to pay, and an upper bound on the marginal cost by looking at the maximum and minimum price. Then, from their estimated search costs, we can conclude that some Goldilocks conditions hold for most of their products.

firms and the consumer act simultaneously, the unique equilibrium is the Diamond paradox.

On the other hand, if the search cost decreases with the number of firms in the market, one could derive a pro competitive effect of entry. For instance, suppose that $s_n \propto 1/n$. In a spatial model, one could think of business clusters where firms concentrate information in a certain area, thus decreasing the search cost per firm. In such an extension, price dispersion can be sustained even as $n \to \infty$.

Finally, note that consumer search is not trivial. The expected number of searches is not limited to one or two, because realized prices can be above the reservation price. Possibly, consumers search for several periods, in contrast with previous approaches such as Burdett and Judd (1983).

4 Concluding remarks

The strongest assumption of this paper regards how the consumer knows the price distribution, and why firms do not react to it. Alternatively, suppose that consumers search from an unknown distribution of which they have an uninformative Dirichlet prior. Rothschild (1974) showed that the optimal search rule will have a reservation price that satisfies properties analogous to those of Lemma 5. Therefore, there is no great loss of making such a simplifying assumption towards accomplishing the goal of this paper. While searching from a known distribution is a strong assumption, one can only expect that relaxing the assumption would reinforce the result rather than reverse it.[13]

On the other hand, firms do not react to prices because they move simultaneously. Yet, consumers do "react" because they move later in the game. Thus, I assume an informational asymmetry. If firms are allowed to react, the issue is commitment as in Daughety (1992). But, if the consumer is prohibited to react, we return to the Diamond setup. A more symmetric informational assumption is that neither firms nor the consumer know the distribution of prices, which implies that firms cannot react to other prices. But as I discussed above, we can approximate the consumer's behavior in the second stage by assuming that she knows the price distribution.

[13] Indeed, using some uninformative prior will assume away the Diamond paradox since consumers will search more than once. Parakhonyak and Sobolev (2015) consider search without priors.

On the plus side, the model offers a novel existence theorem that does not depend on exogenous heterogeneity or stochastic shocks. With minimal assumptions, this paper produces nontrivial consumer search and price dispersion. Finally, by proving that information frictions are sufficient, and not only necessary for price dispersion, we can now think of heterogeneity in models as a tool and not as a necessity.

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A Proofs

Lemma 5 (Properties of the reservation price $p^r(\mathbf{p})$).

1. $p^r(\mathbf{p})$ exists and is increasing in s; in particular, $p^r(\mathbf{0}) = s$.
2. $p^r(\mathbf{p})$ is bounded above by $s + \bar{p}$.
3. $p^r(\mathbf{p})$ is continuous in $\mathbf{p}$.
4. $\mathbf{p} \geq \mathbf{p}^* \Rightarrow p^r(\mathbf{p}) \geq p^r(\mathbf{p}^*)$; where $\geq$ is element-wise.

Proof. Existence comes from the fact that the right hand side of equation (1) is continuous, concave in p^r and equal to $s > 0$ when $p^r = 0$. Note also that the right hand side of equation (1) becomes $s + \frac{1}{n}\sum_{j=1}^{n} p_j$ when $p^r > \max_j\{p_j\}$.

Continuity of $p^r(\mathbf{p})$ is straightforward. Finally, increasing any price will not decrease the right hand side of equation (1). □

Corollary 7. (Symmetric equilibrium) Let $\mathcal{P}_i$ be discrete for all i. Define q_j^i as the symmetric equilibrium probabilities of firm i choosing the jth price in the set $\mathcal{P}_i$, and π as the symmetric equilibrium payoff for all firms. Then, R^* and the mixed strategy $(q_1, \ldots, q_v)$ for all firms are a subgame perfect Nash equilibrium of Game 1 if and only if $(q_1, \ldots, q_v, \pi)$ is the solution of the following system of nonlinear equations.

$$\left[\sum_{k_2=1}^{v} \cdots \sum_{k_n=1}^{v} q_{k_2} \cdots q_{k_n} \pi^* \left(\frac{1}{j}, \frac{1}{k_2}, \ldots, \frac{1}{k_n}, R^*\right) - \pi\right] q_j = 0 \quad j = 1, \ldots, v$$

$$\sum_{k_2=1}^{v} \cdots \sum_{k_n=1}^{v} q_{k_2} \cdots q_{k_n} \pi^* \left(\frac{1}{j}, \frac{1}{k_2}, \ldots, \frac{1}{k_n}, R^*\right) \leq \pi \quad j = 1, \ldots, v$$

$$q_j \geq 0 \quad j = 1, \ldots, v$$

$$\sum_{j=1}^{v} q_j = 1 \tag{3}$$

With $\pi^*\left(\frac{1}{J}, \ldots, R^*\right)$ as defined in equation (2).

Proof. See Theorem 7.1 in Jehle and Reny (2011) for example.

B Calculations in the example

The example has three Nash equilibria. Here I will show that the firms playing $\left\{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$ with probabilities $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)$ is a symmetric Nash equilibrium of the game. Call this strategy $\mathbf{q}$, and note that the consumer does not mind about $\frac{1}{5}$ dollar differences. Then,

$$\begin{aligned}
\pi^*\left(\frac{2}{5}, \mathbf{q}\right) &= \frac{1}{4}\pi^*\left(\frac{2}{5}, \frac{2}{5}\right) + \frac{1}{4}\pi^*\left(\frac{2}{5}, \frac{3}{5}\right) + \frac{1}{2}\pi^*\left(\frac{2}{5}, \frac{4}{5}\right) \\
&= \frac{1}{4}\frac{1}{5} + \frac{1}{4}\frac{1}{5} + \frac{1}{2}\frac{2}{5} \\
&= \frac{3}{10} \\
\pi^*\left(\frac{3}{5}, \mathbf{q}\right) &= \frac{1}{2}\frac{3}{5}\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{2}\right) = \frac{3}{10} \\
\pi^*\left(\frac{4}{5}, \mathbf{q}\right) &= \frac{1}{4}\frac{2}{5} + \frac{1}{2}\frac{2}{5} = \frac{3}{10}
\end{aligned}$$

Finally, $\pi^*(0, \mathbf{q}) = 0$,

$$\pi^*\left(\frac{1}{5}, \mathbf{q}\right) = \frac{1}{4}\frac{1}{2}\frac{1}{5} + \frac{1}{4}\frac{1}{5} + \frac{1}{2}\frac{1}{5} = \frac{7}{40} \text{ and } \pi^*(1, \mathbf{q}) = \frac{1}{2}\frac{1}{2}1 = \frac{1}{4}$$

which are all less than 3/10. Therefore, $\mathbf{q}$ is a Nash equilibrium.

The other equilibria include: playing $\left\{\frac{3}{5}, \frac{4}{5}, 1\right\}$ with probabilities $\left(\frac{1}{5}, \frac{7}{15}, \frac{1}{3}\right)$; and, $\left\{\frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\right\}$ with probabilities $\left(\frac{1}{6}, \frac{1}{6}, \frac{5}{9}, \frac{1}{9}\right)$. These equilibria can be verified in a similar manner. This example was first presented in García P. and Tudón M. (2010).