

CE-370 Reliability of Engineering Systems

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SUMMARY OF LECTURE 6 - CONDITIONAL CDF AND PDF

The conditional cumulative distribution function of a random variable is defined as

$$F_X(x|A) = P(X \leq x|A)$$

which, means by the definition of conditional probability, can be also expressed as

$$F_X(x|A) = \frac{P(X \leq x \cap A)}{P(A)}$$

assuming $P(A) \neq 0$. It is left to the student to show that $F_X(x)$ indeed satisfies all the properties of a CDF.

In analogous fashion, the conditional probability density function can be defined as

$$f_X(x|A) = \frac{dF_X(x|A)}{dx}$$

An important problem in reliability: Consider a system with random lifetime X described by a continuous CDF $F_X(x)$. Find the conditional CDF and PDF given the event A defined as $X \geq t$, that is, the system has not failed at time $x = t$.

We begin by first defining

$$F_X(x|A) = \frac{P(X \leq x \cap X > t)}{P(X > t)}$$

and noting that

$$P(X \leq x \cap X > t) = \begin{cases} 0 & \text{if } x \leq t \\ F_X(x) - F_X(t) & \text{if } x > t \end{cases}$$

and

$$P(A) = P(X > t) = 1 - F_X(t)$$

Combining these results we obtain

$$F_X(x|X > t) = \begin{cases} 0 & \text{if } x \leq t \\ \frac{F_X(x) - F_X(t)}{1 - F_X(t)} & \text{if } x > t \end{cases}$$

Similarly, the conditional PDF is given by

$$f_X(x|X > t) = \frac{dF_X(x|X > t)}{dx} = \begin{cases} 0 & \text{if } x \leq t \\ \frac{f_X(x)}{1 - F_X(t)} & \text{if } x > t \end{cases}$$

The result is depicted in the figure below

