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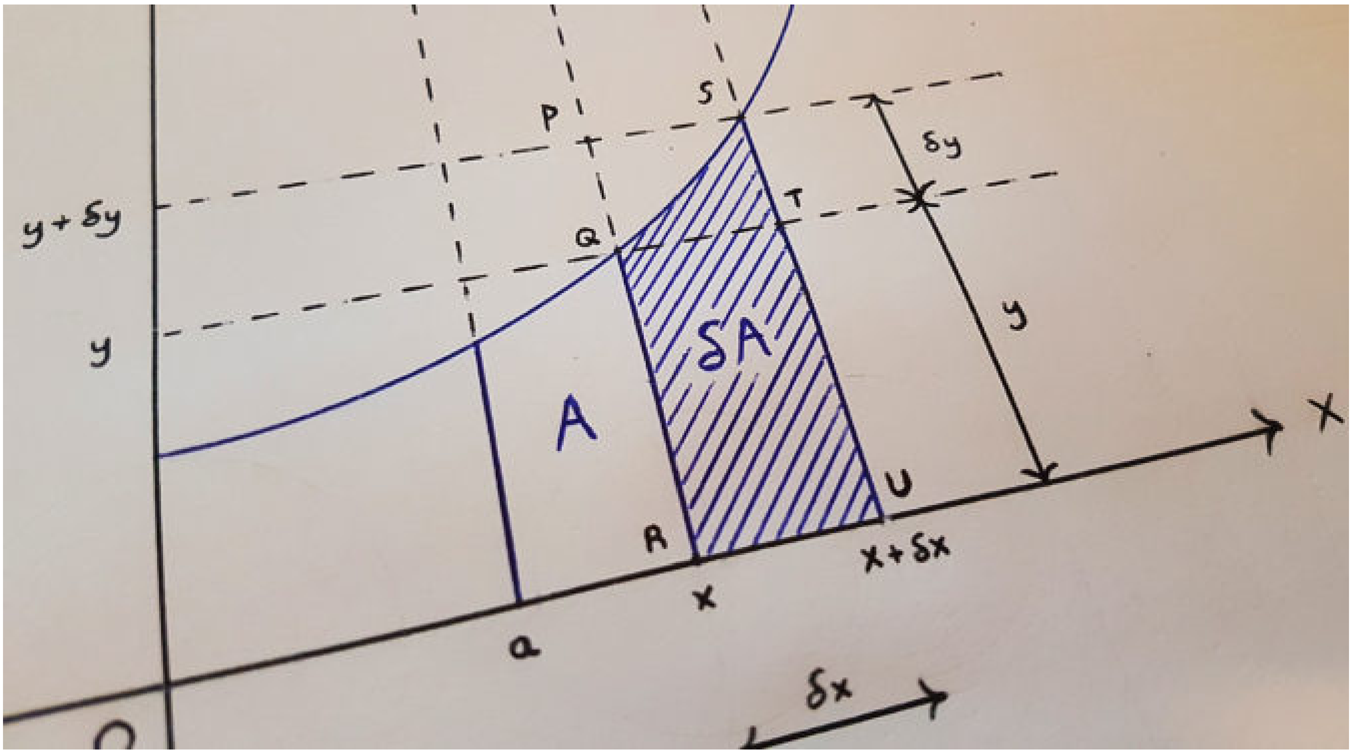
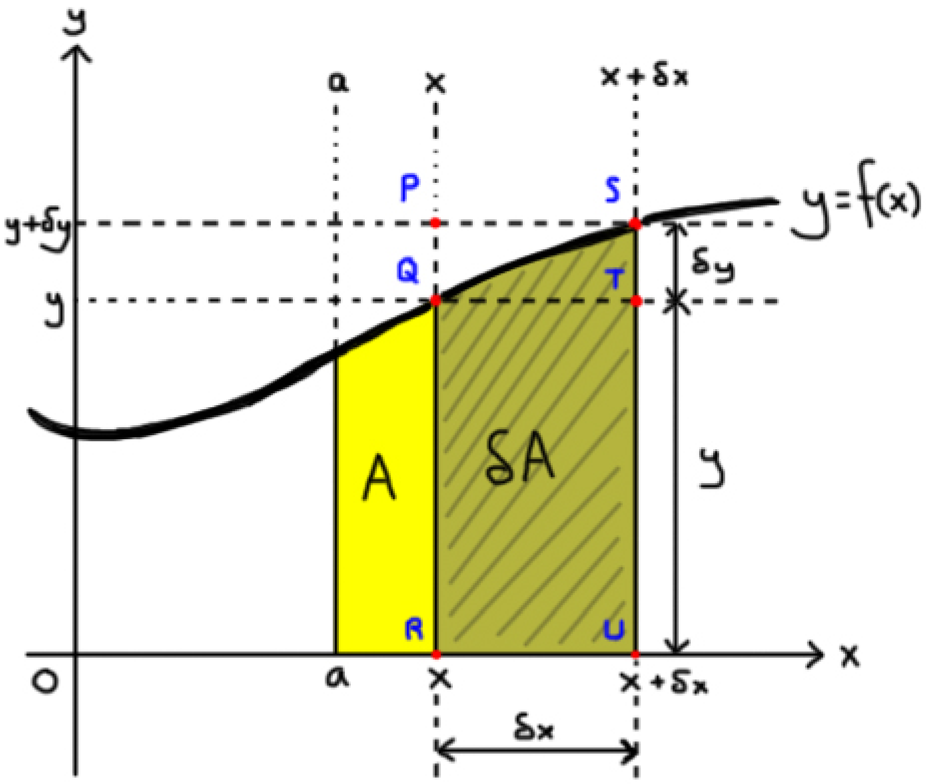


AREAS, CALCULUS, INTEGRATION, VISUALISING MATHEMATICS

DERIVE THE FORMULA TO FIND AREAS UNDERNEATH CURVES

OCTOBER 9, 2016 | TIAGO HANDS | LEAVE A COMMENT

In this post I'll be revealing how you can derive the formula which can be used to **find areas underneath curves**, from absolute scratch. Now, just below, what you will find is the diagram that will help us produce this formula…



In this diagram what you will discover is that:

*Please read the following contents carefully…

- A length **a** exists, which starts at the origin **O** and ends at **a**;
- A length **x** exists, which starts at the origin **O** and ends at **x**;
- A length **x+δx** exists, which starts at the origin **O** and ends at **x+δx**;
- A length **δx** exists, which starts at **x** and ends at **x+δx**;
- A height **y** exists, which starts at the origin **O** and ends at **y**;
- A height **y+δy** exists, which starts at the origin **O** and ends at **y+δy;**
- A height **δy** exists, which starts at **y** and ends at **y+δy;**
- There is a curve called **y=f(x)**;
- There is an area underneath the curve called **A** which commences at **a** and ends at **x**;
- There is an area underneath the curve called **δA** which commences at **x** and ends at **x+δx** (Note: If you extend the distance from a to x what you get is a larger area, and the change in area can be measured. This change or difference is called **δA**);
- There is a rectangle that exists called **QRUT**. It has an area which is **yδx**;
- There is a rectangle that exists called **PRUS**. It has an area which is **(y+δy)δx**;
- **δA** has an area larger than that of the rectangle **QRUT**, but smaller than that of the rectangle **PRUS**.

Producing the formula with the information we've discovered…

Ok, so we want to produce the formula which will help us find areas underneath curves from absolute scratch. At our disposal we have a helpful diagram (which we've looked at and analysed carefully) and we've been able to discover a few facts about it. I think we can now get to work…

Let's start off by saying that:

Area QRUT < δA < Area PRUS

Which is something we already discovered.

If this is the case, we can say that:

yδx < δA < (y+δy)δx

Now, check out what happens when we divide all the elements of this expression by **δx**:

What we end up with is…

$$y < \frac{\delta A}{\delta x} < y + \delta y$$

Alright, now you may be saying to yourself, why do I need to know this? Well, it turns out that:

$$\lim_{\delta x \to 0}\left(\frac{\delta A}{\delta x}\right) = y$$

This is because as δ**x** approaches **0**, δ**y** approaches **0** leaving **(δA)/(δx)** sandwiched between **y** and **y+0.000000000000000001** which is virtually y.

And, also…

$$\lim_{\delta x \to 0}\left(\frac{\delta A}{\delta x}\right) = \frac{dA}{dx}$$

As a consequence, this ultimately means that:

$$y = \frac{dA}{dx}$$

This is incredibly significant, because if we then integrate both sides of this equation, we get:

$$\int ydx = \int \frac{dA}{dx}dx \quad \Rightarrow \quad A = \int ydx$$

And…

$$A = \int ydx = F\left(x\right) + C$$

Now, this equation can actually be used to find the area **A** underneath the curve from a to x. What we're basically saying is that this area is equal to some function of x plus a constant. This 'some function of x' occurs when we integrate y which is a function of x.

Finalising the formula…

Alright so we've managed to latch on to something incredibly significant… We've got an important equation:

$$A = \int ydx = F\left(x\right) + C$$

However, it is not complete. We need to know what the constant C is. So…

If we say that at **x=a** the area **A** underneath the curve is **0**, watch what happens… Look at what we get…

$$O = F\left(a\right) + C$$

Which means that:

$$C = -F\left(a\right)$$

Hence, we can conclude that:

$$A = \int ydx = F\left(x\right) - F\left(a\right)$$

And this formula can be transformed into something more fancy if we are measuring an area underneath a curve from **x=a** to **x=b**…

This is probably the formula you're most familiar with…

$$A = \int_a^b ydx = F\left(b\right) - F\left(a\right) = \left[F\left(x\right)\right]_a^b$$

Which is the formula which can be used to find areas underneath curves.

AREAS CALCULUS FEATURED FUNCTIONS INTEGRATION