LOGARITHMIC REALMS

Brilliant Logarithmic Proofs

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LOGARITHMIC REALMS – *Brilliant Logarithmic Proofs*

In this small booklet I'll be providing you with logarithmic proofs that most maths students have never come across. These logarithmic proofs have been created in order to help you understand logarithms and how they can be manipulated with prolific effect.

1. To begin with, here are a few logarithmic axioms and indices you need to know to understand the logarithmic proofs that I am about to present.

$a^x = n \Rightarrow \log_a n = x \ \{n>0\}$ $\therefore \ \log_a a = 1$ $\therefore \ \log_a 1 = 0$ $m^n = q \Rightarrow m = q^{\frac{1}{n}} \ \{n \neq 0\}$ $\therefore \ a^x = b^p \Rightarrow (a^x)^{\frac{1}{p}} = b \Rightarrow a = (b^p)^{\frac{1}{x}} \{x \neq 0, \ p \neq 0\}$ $(a^m)^n = a^{mn}$ $a^m \cdot a^n = a^{m+n}$ $\frac{a^m}{a^n} = a^{m-n}$	These definitions and mathematical rules must be memorised so that you can manipulate logarithms seamlessly.

PROOF 1:

$$\log_a(b^p) = p \cdot \log_a b$$

PROOF:

Say that:

$$\log_a(b^p) = q$$

Now:

$$a^q = b^p \Rightarrow (a^q)^{\frac{1}{p}} = b \Rightarrow a^{\frac{q}{p}} = b$$

$$\therefore \ \log_a b = \frac{q}{p}$$

$$\therefore \ p \cdot \log_a b = q$$

$$\therefore \ \log_a(b^p) = p \cdot \log_a b$$

PROOF 2:

$$\log_a x = \frac{1}{\log_x a}$$

PROOF:

$$\log_a x = p$$

$$a^p = x$$

$$x^{\frac{1}{p}} = a$$

$$\log_x a = \frac{1}{p}$$

$$p \cdot \log_x a = 1$$

$$p = \frac{1}{\log_x a}$$

$$\therefore \; \log_a x = \frac{1}{\log_x a}$$

PROOF 3:

$$\log_a x = \frac{\log_b x}{\log_b a}$$

PROOF:

Say that:

$$\log_b x = p$$

$$\therefore \; b^p = x$$

Therefore:

$$\frac{\log_b x}{\log_b a}$$

$$= \log_b x \cdot \log_a b$$

$$= \log_b (b^p) \cdot \log_a b$$

$$= p \cdot \log_b b \cdot \log_a b$$

$$= p \cdot \log_a b$$

$$= \log_a (b^p)$$

$$= \log_a x$$

$$\therefore \; \log_a x = \frac{\log_b x}{\log_b a}$$

PROOF 4:

$$\log_a(xp) = \log_a x + \log_a p$$

PROOF:

Say that:

$$a^m = x$$

$$\therefore \log_a x = m$$

$$a^n = p$$

$$\therefore \log_a p = n$$

This means that:

$$\log_a(xp)$$

$$= \log_a(a^m \cdot a^n)$$

$$= \log_a(a^{(m+n)})$$

$$= (m+n) \cdot \log_a a$$

$$= m+n$$

$$= \log_a x + \log_a p$$

PROOF 5:

$$\log_a\left(\frac{x}{p}\right) = \log_a x - \log_a p$$

PROOF:

Say that:

$$a^m = x$$

$$\therefore \ \log_a x = m$$

$$a^n = p$$

$$\therefore \ \log_a p = n$$

This means that:

$$\log_a\left(\frac{x}{p}\right)$$

$$= \log_a\left(\frac{a^m}{a^n}\right)$$

$$= \log_a\left(a^{(m-n)}\right)$$

$$= (m-n) \cdot \log_a a$$

$$= m - n$$

$$= \log_a x - \log_a p$$

Thankyou for downloading this file. I hope this small booklet has been very useful to you.

Best wishes,

Thiago De Carvalho